Pointwise Defined Sets in Lebesgue Spaces and Their Clarke Cones.

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Abstract

Let (T, μ) be a complete $\sigma-$finite measure space, E be a separable Banach space, and $S : T \to 2^E$ be a measurable multivalued mapping with closed values. We study the Clarke tangent cone to the sets $H_p(S) := \{z \in L^p(T, E) : z(t) \in S(t) \quad a.e.\}$ where $1 \leq p \leq \infty$.

Páles and Zeidan [3] have studied the case $p = \infty$ whenever the underlying measure space is finite and $E = \mathbf{R}^n$.

Mehlitz and Washsmuth [2] have studied the problem whenever $p \in (1, \infty)$ assuming geometric regularity (a derivability) of the sets $S(t)$.

Giner [1] has studied a similar problem for epigraph of an integral functional under a suitable growth condition on the integrand.

We prove that whenever $1 \leq p < \infty$ and E is reflexive, it holds that

$$\widehat{T}_{H_p}(x) = \{v \in L^p : v(t) \in \widehat{T}_{S(t)}(x(t)) \quad a.e.\}.$$

We make use of a Theorem of Borwein and Strojwas about weak Bouligand tangent cone.

If $p = \infty$, then it holds $\widehat{T}_{H_\infty}(x) \subset \{v \in L^\infty : v(t) \in S(t) \quad a.e.\}$. For the reverse inclusion only partial results are true.

As an application, a Lagrange multiplier rule for the problem (**IP**) is obtained, where

$$(\mathbf{I}P) \qquad f(x) := \int_T \varphi(t, x(t)) d\mu(t) \to \min \qquad \text{subject to} \qquad x \in H_p(S),$$

with $\varphi : T \times \mathbf{R}^n \to \mathbf{R} \cup \{+\infty\}$ measurable in $t \in T$ and proper lower semicontinuous in $u \in \mathbf{R}^n$.

References

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[3] Zs. Páles and V. Zeidan, On L^1-closed decomposable sets in L^∞, *Journal of Mathematical Analysis and Applications* **238** (1999), no. 2, 491–515.